

CONTRAIL DISTANCE FORMATION MODEL

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A model has been developed to estimate the distance behind the engines that a contrail, i.e., condensed trail of water vapor, is expected to form. The results of this model agree exceptionally well with a [statement issued by the United States Air Force](#) that "contrails become visible roughly about a wingspan distance behind the aircraft". There is now an abundance of photographic and video evidence that consistently and visibly demonstrates the repeated formation of aerosol trails in much closer proximity to the engines than that which is established by the Air Force, as well as that which is predicted from the model described below. These trail formations are in direct contradiction to a statement of fact issued by the United States Air Force. This evaluation now adds to the multitude of studies which

multitude of studies which conclusively demonstrate that the emissions from these aerosol operations are not composed primarily of water vapor. This model is not intended to encompass all variables that may be in effect, but does represent a rational attempt to model the physics of contrail formation times involved. Any corrections to this study will be made as is appropriate. This model is in addition to that [previously developed](#) related to expected contrail dissipation times, as well as originating [relative humidity studies](#) at flight altitude.

The model is developed as follows:

Let us assume that the temperature of the exhaust emissions of the aircraft is approximately 1000 deg. C., which is an apparent reasonable estimate (see [Principles of Jet Engine Operation](#)). The model can easily be generalized to encompass any reasonable ranges in temperature that are expected within the combustion process and subsequent exhaust emissions. The model is not highly sensitive to expected changes in temperature at this level, and if a more accurate

value becomes available, it will be used in the future.

Let us assume the temperature of the atmosphere at flight altitude, approximately 35,000 ft. MSL is -50 deg. C. Again, each variable within the model can be generalized as needed, and the sensitivity of the model to these changes can be evaluated.

The amount of heat extraction required to cool the exhaust vapor can be given as follows:

$$H = dH(\text{ice}) + dH(\text{melting}) + dH(\text{water}) + dH(\text{vap}) + dH(\text{steam})$$

for the sake of initial example and simplicity, and to demonstrate numerical results, let us apply this to 1 gram of water:

$$-H = (1 \text{ gm}) (.5 \text{ cal / (gm * K)}) (50 \text{ deg. K})$$

$$+ (1 \text{ gm}) (80 \text{ cal / gm})$$

$$+ (1 \text{ gm}) (1.0 \text{ cal / (gm * K)}) (100 \text{ deg. K})$$

$$+ (1 \text{ gm}) (540 \text{ cal / gm})$$

$$+ (1 \text{ gm}) (.33 \text{ cal / gm}) * 900 \text{ deg. K}$$

or

$H = -(25 + 80 + 100 + 540 + 300)$
cal. = -1045 cal. required to cool
steam at 1000 deg. C. to 1 gm of
ice at -50 deg. C.

Now,

1 calorie (cal) = 4.1868 Joules (J)

Therefore,

-1045 cal = -4375 J.

Next, to consider a realistic
particle size for emissions from
aircraft, the Max Planck
Institute has stated that the
average size of particles emitted
from aircraft is approximately 30
to 200 microns in size. As a side
note, the average particle size of
cloud nuclei is stated by Vincent
Schaefer, *Atmosphere*, to be
from 0.2 to 0.3 microns. Let us
assume an average size of 115
microns on each side of a cube
particle.

Since 1 gm. of water = 1 cu. cm
in volume, a cube particle size of
115 microns in dimension on
each side has a volume of:

$(115 \times 10^{-6})^3$ meters, or 1.52×10^{-12}
cu. m.

Since 1 gm. of water has a volume of $(1\text{E}-2)^3$ meters, the volume of a gram of water is $(1\text{E}-6)$ cu. m.

The ratio in volume of a particle of dimension 115 microns to a gram of water is:

$$1.52\text{E}-12 \text{ cu. m.} / 1 \text{ E}-6 \text{ cu. m}$$

or

$$(1.52\text{E}-6)$$

The amount of heat required to cool the 115 micron particle is therefore

$$(1.52\text{E}-6) (4375 \text{ J}) = 6.654\text{E}-3 \text{ J.}$$

for a particle 115 microns thick and corresponding to a temperature change of 1050 deg. C. [note units are therefore: J / (m * K)]

Now evaluate the thermal conductivity of the medium in which the particle exists, i.e., air. From the REA Handbook of Mathematical, Scientific, and Engineering Formulas, Tables, Functions, Graphs, and Transforms, the thermal conductivity of air at - 50deg. C. is given as .012 Btu / (hr * ft * deg. F).

Converting this value to SI units,

$$.012 \text{ Btu} / (\text{hr} * \text{ft} * \text{deg F.}) \rightarrow \\ (1055 \text{ J} / \text{Btu}) / ((3600 \text{ sec/hr}) * \\ (.3048 \text{ m/ft.}) * ((5/9) \text{ deg. K} / \text{deg. F}))$$

or the thermal conductivity of air at a temperature of -50 deg. C can be given as

$$.02075 \text{ J} / (\text{s} * \text{m} * \text{K})$$

Therefore the amount of time required to cool the particle from 1000 deg. C to -50 deg. C is given by:

$$(6.654 \text{E-}3 \text{ J} / (\text{m} * \text{K})) / (.02075 \text{ J} / (\text{s} * \text{m} * \text{K})) = .321 \text{ seconds.}$$

Now for an aircraft traveling at 500 mph, this translates to approx. 733 ft./sec.

Therefore, the particle evaluated will cool to the ambient temperature in approximately:

$$(733 \text{ ft./sec}) * .321 \text{ sec} = 235 \text{ feet behind the engines of the aircraft.}$$

A Boeing 757 measures approximately 155 ft. in length. The distance from the rear of the engines to the tail of the aircraft

is approximately 80 feet (scaled). Therefore the contrail is expected to form approximately (235 ft. - 80 ft.), or approximately 155 ft. behind the tail of the aircraft. The wingspan of a Boeing 757, being used as a representative example, is approximately 125 feet in width. The results of this model agree quite well (approx. 30 ft. coupled with the transition zone) therefore, with the expected physics and chemistry of water vapor as well as with the statement provided by the United States Air Force. The model will show itself to be sensitive to particle size. "Contrail" formation in front of, or immediately adjacent to the stabilizer region of the aircraft, is not to be expected either from the results of this model, or from that statement issued by the Air Force.

Significant deviations from these results as well as from the USAF statement, as they occur repeatedly in conjunction with the aerosol operations, are tangible evidence of non-water vapor emissions that are involved.

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